American International Journal of Research in Science, Technology, Engineering & Mathematics

Available online at http://www.iasir.net

ISSN (Print): 2328-3491, ISSN (Online): 2328-3580, ISSN (CD-ROM): 2328-3629

AIJRSTEM is a refereed, indexed, peer-reviewed, multidisciplinary and open access journal published by International Association of Scientific Innovation and Research (IASIR), USA
(An Association Unifying the Sciences, Engineering, and Applied Research)

28-Day Strength Predicting Model for Foamed Aerated Concrete Containing Pulverized Bone as a Partial Replacement of Cement

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Abstract: This paper presents the results of a mathematical model developed for predicting the 28-day compressive strength of foamed aerated concrete containing pulverised bone as partial replacement of cement for the purpose of quality controls. Strength-porosity relation was used as a basis for the development of the model, and the Bisection Methods of Numerical analysis was used to evaluate the inherent constants. For all the percentages of cement replacement with pulverised bone, the experimental strength values compare well with those of the model. It can thus be concluded that the model is valid for foamed aerated concrete with and without pulverized bone. The model has been validated up to 20% replacement of cement with pulverised bone.

Keywords: Bisection Method, Model, Porosity, Pulverised bone, Compressive strength.

I. Introduction

Concrete is a composite engineering material that has become the premier, most favoured, and versatile of all the construction materials of building and civil engineering construction. This is because it can be produced in a variety of strengths, stiffnesses, unit weights, porosities and durability characteristics and properties, by using the same four basic components of cement, fine sand, coarse aggregates, and water. Recent innovations includes, either alteration in its composition during mix designs, or addition of chemical admixtures like superplasticizers, or adding of mineral additives like fly ash, granulated blast furnace, silica fume, pulverised bone, etc. to produce concretes of diverse and improved properties specialized applications. One of these types of concrete is foamed aerated concrete which is now gradually becoming a structural concrete. But in structural concrete, strength is the primary criterion in the selection of concrete. Structural concrete used in construction gains strength over time. The strength of concrete used in design is the characteristic compressive strength, which is the strength gained by the concrete over a period of 28 days. The time lag between casting and the determination of the strength on the 28-day is such that if the strength falls short of the specified strength, large amount of money will be required to pull down the structure and rebuild it. On the other hand, if the strength is excessive, a lot of resources would have been wasted. None of these situations is desired by an engineer. However, laboratories with compression-testing machine are not only too far from sites, but also charged colossal amount of money to carry out such tests. Thus an easy, rapid, cheap, and reliable means of predicting the 28-day strength of concrete will be of great significance. Several strength predicting relations exist for plain cement paste, mortar, and concrete. And it is usually based on Abram's Law for well compacted concrete, in which the only parameter to be evaluated is the water-cement ratio. However, in concrete with deliberately entrapped air voids, like the foamed aerated concrete, an expression developed by Feret, that included the volume of air voids has been found to be a better alternative [1]. Notable researchers have developed expressions for predicting the strength of foamed concrete [2, 3]. Researchers have however compared strength-porosity and gel-space ratio strength predicting mathematical models, using foamed concrete incorporating fly ash [4]. They found out that expressions derived from strength-porosity model correlates well with the measure strength. Recent works by [5,6,7] have shown that pulverised bone up to 20% can be used to replace cement in the production of foamed aerated concrete. The present study is concerned with developing and validating a 28-day compressive strength prediction model for foamed aerated concrete containing pulverised bone as partial replacement of cement using the strength-porosity expression and employing the properties of the constituents, in addition to parameters like: the fresh density and the compressive strength.

II. Materials and Methodology

The material constituents and proportion that were used for the production of foamed aerated concrete are presented in Table 1.

Table 1: Mix Constituent Proportions for the Foam Concrete Mixes

% PB*	Binder (kg)		Sand (kg)	Water for Base Mix (kg)	Foam Concentration	
	Cement	PB*			Mixing Water	Foam (g)
0%	25.00	0.00	75	12.50	4.688	187.5
5%	23.75	1.25	75	12.50	4.688	187.5
10%	22.50	2.50	75	12.50	4.688	187.5
15%	21.25	3.75	75	12.50	4.688	187.5
20%	20.00	5.00	75	12.50	4.688	187.5

*PB = Pulverized Bone

In order to determine the fresh density, a standard container of known volume was used. The compressive strength at 28 days was determined with the means of the Avery Universal Testing machine.

III. The 28th Day Strength-Predicting Model: Development

Expression relating the porosity and the compressive strength of foamed concrete with cement paste was developed by [7]. He used a simple model in which foamed aerated concrete is composed of air, evaporable water, non-evaporable water, and cement (figure 1).

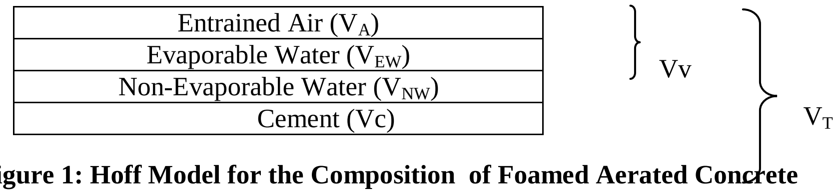

Figure 1: Hoff Model for the Composition of Foamed Aerated Concrete

But the foamed aerated concrete used in this work contained sand and pulverised bone. Thus, in order to account for this, the Hoff model has been expanded to include sand and binder (cement and pulverised bone). The modified form of the Hoff model used for this work, for the purpose of strength-prediction is shown in figure 2. Because of high porosity of foamed aerated concrete, equation expressing relationship between strength and porosity has been considered a better tool in predicting the strength of foamed aerated concrete containing fly ash [4].

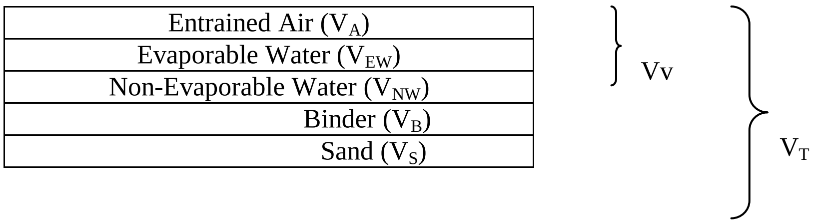

Figure 2: Modified form of Hoff Model for the Composition of Foamed Aerated Concrete

From Figure 2,

V_A = volume of air entrained in the concrete through aeration process

V_{EW} = volume of evaporable water

V_{NW} = volume of non-evaporable water.

$= 0.20\rho_B V_B$ [4] **(1)**

where: ρ_B = specific gravity of binder, and V_B = volume of binder

V_B = volume of binder. The combination of cement and pulverised bone. But in the control specimens, it is only the volume of cement.

V_S = volume of sand

This has been used as the basis for the development and validation of strength-prediction for the present work. The equation is expressed as:

$$f_c = f_o(1 - p)^n \quad \textbf{(2)}$$

where

f_c = compressive strength at 28th day

f_0 = intrinsic compressive strength at zero porosity

p = porosity

n = empirical constant depending on the properties of the material.

Using a multi-phase weight-volume relationships [8, 9], the theoretical porosity, p for foamed concrete with Vv representing the volume of all the voids, and total volume V_T, can be expressed as:

$$p = \frac{V_V}{V_T} \quad \textbf{(3)}$$

$$= \frac{V_A + V_{EW}}{V_B + V_W + V_A + V_S}$$

where:

V_{EW} = volume of non-evaporable water
V_B = volume of binder (cement + pulverised bone)
V_W = volume of evaporable and non-evaporable water (V_{EW} + V_{NW})

In this expression the volume of water V_W is considered to compose of both evaporable and non-evaporable water. Also V_B represents the volume of the binder which consists of cement and pulverised bone. Other parameters are as defined in Figure 2.

Also if W_T is the total weight of the constituent materials and V_T is the total volume, then the wet density (d_c) of foamed concrete, in relation to Figure 1 can be expressed as:

$$d_c = \frac{W_T}{V_T} \quad \textbf{(4)}$$

$$= \frac{W_B + W_W + W_S}{V_B + V_{NW} + V_S + V_V} \quad \textbf{(5)}$$

where:

W_B = weight of binder (cement and pulverised bone)
W_W = weight of water
W_S = weight of sand
V_B = volume of binder (Cement and pulverised bone)
V_{NW} = volume of non-evaporable water
V_S = volume of sand
V_V = volume of all voids

Now if k_{ws} is water/solid ratio by weight (solid being binder and sand), then

$$k_{ws} = \frac{W_W}{(W_B + W_S)}$$

$$W_W = k_{ws}(W_B + W_S) \quad \textbf{(6)}$$

Putting equation 6 into equation 5, we have:

$$d_c = \frac{(W_B + k_{WS}(W_B + W_S) + W_S)}{(V_B + V_{NW} + V_S + V_V)}$$

$$= \frac{(W_B + k_{WS}W_B + k_{WS}W_{S} + w_S)}{(V_B + V_{NW} + V_S + V_V)}$$

$$= \frac{(W_B + k_{WS}W_B + W_S + k_{WS}W_S)}{(V_B + V_{NW} + V_S + V_V)}$$

$$= \frac{\{W_B(1 + k_{WS}) + W_S(1 + k_{WS})\}}{(V_B + V_{NW} + V_S + V_V)}$$

$$d_c = \frac{(1 + k_{WS})(W_B + W_S)}{(V_B + V_{NW} + V_S + V_V)} \quad \textbf{(7)}$$

From equation 4

$$W_T = d_c V_T \quad \textbf{(8)}$$

$$V_T = V_B + V_{NW} + V_S + V_v \quad \textbf{(9)}$$

Substituting for the values of d_c and V_T in equation 8, we have:

$$W_T = \frac{(1 + k_{WS})(W_B + W_S)}{(V_B + V_{NW} + V_S + V_V)} \times (\mathbf{V_B + V_{NW} + V_S + V_v})$$

$$W_T = (1 + k_{WS})(W_B + W_S) \quad \textbf{(10)}$$

Substituting for W_T and V_T in equation 8,:

$$d_c(V_B + V_{NW} + V_S + V_V) = (1 + k_{WS})(W_B + W_S)$$

$$d_cV_B + d_cV_{NW} + d_cV_S + d_cV_C = (1 + k_{WS})(W_B + W_S)$$

$$d_cV_V + d_c(V_B + V_{NW} + V_S) = (1 + k_{WS})(W_B + W_S)$$

substituting for the value of V_{NW} (equation 1)

$$V_V = \frac{1}{d_C}\{(1 + k_{WS})(W_B + W_S) - d_C(V_B + 0.2V_B\rho_B + V_S\} \quad \textbf{(11)}$$

Now, recalling the experession for the porosity p:

$$p = \frac{V_v}{V_T} \quad \text{(from equation 3)}$$

$$= \frac{V_V}{\frac{W_T}{d_C}} \quad \text{(by replacing } V_T \text{ by } \frac{W_T}{d_c} \text{ in equation 4)}$$

By combining equations 8 for W_T, and 11 for V_V, the expression for porosity p becomes:

$$p = \frac{\frac{1}{d_C}\{(1 + k_{WS})(W_B + W_S) - d_C(V_B + 0.20V_B\rho_B + V_S)\}}{\frac{\{(1+k_{WS})(W_B+W_S)\}}{d_C}} \quad \textbf{(12)}$$

Simplification yields:

$$p = 1 - \frac{d_C(V_B + 0.20V_B\rho_B + V_S)}{(1+k_{WS})(W_B + W_S)} \quad \textbf{(13)}$$

Now if sand/binder ratio weight is represented by as k_{SBW}, then:

$$k_{SBW} = \frac{W_S}{W_B} \quad \textbf{(14)}$$

so that

$$W_S = k_{SBW}W_B \quad \textbf{(15)}$$

Similarly representing sand/binder ratio by volume as k_{SBV}, gives:

$$k_{SBV} = \frac{V_S}{V_B} \quad \textbf{(16)}$$

so that

$$V_S = k_{SBV}V_B \quad \textbf{(17)}$$

Substituting equations 15 and 17 for Ws and Vs in equation 13 then,

$$p = 1 - \frac{d_C(V_B + 0.20V_B\rho_B + k_{SBV}V_B)}{(1+k_{WS})(W_B + k_{SBW}W_B)}$$

$$= 1 - \frac{d_C V_B(1+0.20\rho_B + k_{SBV})}{(1+k_{WS})(1+k_{SBW})W_B}$$

$$= 1 - \frac{d_C V_B(1+0.20\rho_B + k_{SBV})}{(1+k_{WS})(1+k_{SBW})\rho_B\gamma_W V_B}$$

$$\mathbf{p} = 1 - \frac{d_C(1+0.20\rho_B + k_{SBV})}{(1+k_{WS})(1+k_{SBW})\rho_B\gamma_W} \quad \textbf{(18)}$$

Where γ_W is the unit weight of water. Now substituting equation 18 into the strength-porosity expression of equation 2

$$f_c = f_o(1 - p)^n$$

the strength equation becomes:

$$f_C = f_0\left(\frac{d_C(1+0.20\rho_B + k_{SBV})}{(1+k_{WS})(1+k_{SBW})\rho_B\gamma_W}\right)^n \quad \textbf{(19)}$$

where:

f_c = the compressive strength at 28-day

f_o = the compressive strength at zero porosity

d_c = wet density of the foamed aerated concrete

ρ_B = specific gravity of the binder

k_{SBV} = sand/binder ratio by volume

k_{WS} = water/solid ratio by weight

k_{SBW} = sand/binder ratio by weight

Careful observation of this equation shows that: i) the equation depends on the physical properties of the constituent materials alone, and ii) these physical properties are easily measurable in the laboratory, thus making it easy for use as quality and measure. Using the Bisection Method of Numerical Analysis [10, 11], the values of the model constants f_o and n were found through the process of successive approximation to be 104.55N/mm^2 and 3.20 respectively (the underlying principle guarding the usage of the method is contained in Appendix 1). The equation 18 now becomes:

$$f_C = 104.55\left(\frac{d_c(1+0.20\rho_B+k_{SBV})}{(1+k_{WS})(1+k_{SBW})\rho_B\gamma_W}\right)^{3.20} \quad \textbf{(20)}$$

where:

f_c = the compressive strength at 28day
d_c = fresh density of the foamed aerated concrete
ρ_B = specific gravity of the binder
k_{SBV} = sand/binder ratio by volume
k_{WS} = water/solid ratio by weight
k_{SBW} = sand/binder ratio by weight

This is the 28-day strength-predicting equation for foamed aerated concrete, in less than 3hrs after production in terms of wet density, specific gravity of the binder, sand/binder ratio by volume, sand/binder ratio by weight, and water/solid ratio by weight, all of which are easily measurable in the laboratory.

IV. Validation and Discussion of Results

In order to validate the derived mathematical strength-predicting model, the values of the compressive strengths obtained from using equation 19 taking into consideration the physical properties in Appendices 2 and 3, are compared with the compressive strengths obtained from experimental investigation through the cube tests for all the levels of cement replacement with pulverized bone. These results are presented in Table 2 and Appendix 4.

Table 2: Comparison of the Model and Experimental Compressive Strengths

	Model (N/mm^2)	Experimental (N/mm^2)	% Difference
0%	15.43	15.43	0.00
5%	14.55	14.23	+2.20
10%	13.70	14.01	- 2.92
15%	13.20	13.26	- 0.46
20%	12.89	12.98	- 0.70
Average	13.95	13.98	-1.05

From this Table 2, it can be seen that the values of compressive strength obtained from the model equation 20 compare very well with the values of experimental compressive strength for all the levels of cement replacement with pulverised bone. When expressed as a percentage of model strength, it can be seen that the differences between the predicted and experimentally observed strengths for all the replacement level varies between 0% (for the control) and - 2.92% (for 10% replacement of cement). The highest difference – 2.92% occurred at 10% replacement level. The overall average difference is -1.05%. The average being less than 10% is considered to be acceptable for laboratory-produced foamed aerated concrete [12].

Also from statistical analysis, the mean is 13.95N/mm^2, and the standard deviation is 1.04. The lower value of standard deviation is an indication of the data clustering around the average. Further, the statistical significant test at significance levels of 1%, 5%, and 10% showed that the difference is not significant, as the confidence values fall outside the critical regions, indicating that there is no reason to reject the result of the model at the level of confidence of 1%, 5%, and 10%. Thus the strength-predicting equation 20 can be considered valid for foamed aerated concrete with and without pulverised bone as replacement for cement, provided the level of replacement does not exceed 20%.

The curves for the experimental and predicted strength are shown in Figure 3. It can be observed that the predicted and experimental strengths are the same at for the control (0% replacement). At 5% replacement, the model strength is higher than experimental strength. From 10% replacement level upward, the model strengths are lower than the experimental strengths.

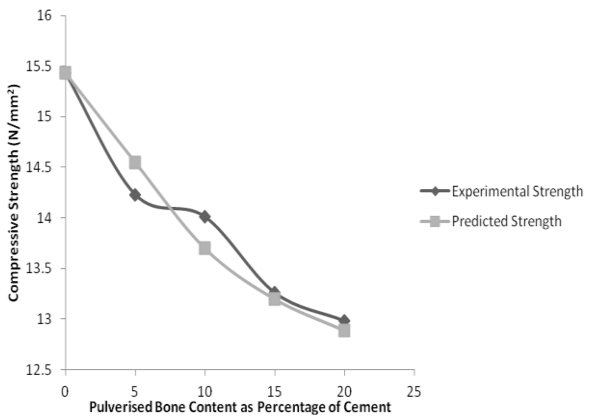

Figure 3: Variation of Experimental and Model Strengths with Pulverised Bone

The relationship between the experimental and the predicted compressive strength values can be represented by a scatter plot in Fig. 4.

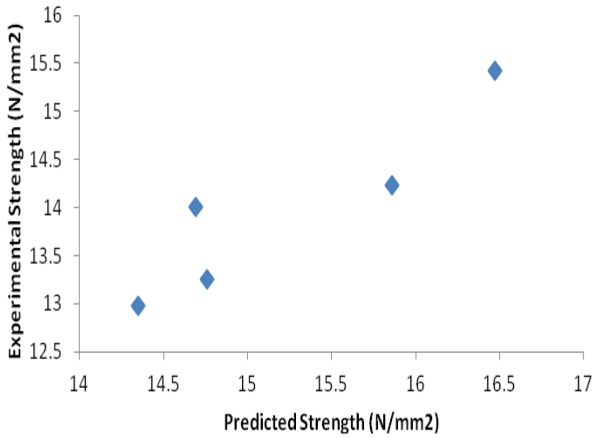

Figure 4: Relationship between the Experimental and Predicted Strength Values

A correlation coefficient of 0.977 indicates a strong and positive linear relationship between the model strength and experimental strength. Using the statistical line of best fit, this relationship can be expressed through a linear regression equation of the form:

$$f_{cue} = Af_{cum} + B \quad \textbf{(21)}$$

where:

f_{cue} = experimentally observed compressive strength

f_{cum} = compressive strength obtained from the mathematical model

A and B = regression coefficients representing the slope and intercepts respectively of a plot of experimental compressive strength against model compressive strength.

By determining these coefficients through regression analysis, equation 21 becomes:

$$f_{cue} = 1.06f_{cum} - 0.80 \quad \textbf{(22)}$$

where:

f_{cue} = experimentally observed compressive strength

f_{cum} = compressive strength obtained from the mathematical model

V. Conclusions

From the foregoing, the following conclusions are made.

i) A mathematical model for predicting the 28-day compressive strength of foamed aerated concrete with and without pulverized bone has been developed and validated.

ii) The model has been validated up to 20% partial replacement of cement with pulverised bone.

iii) The model predicts 28th day compressive strength from freshly-mixed concrete, thus it could be used as an effective quality control measure on construction sites.

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Appendix 1 – Numerical Analysis: The Bisection Method

The Bisection Method is a method that is based on intermediate value theorem, for the purpose of finding the root of a non-linear equation of the form:

$$f(x) = 0 \qquad \textbf{1}$$

Given that a root of equation 1 exists within an open interval (a,c), then by the rule,

$$f(a)*f(c) < 0 \qquad \textbf{2}$$

The first step in the bisection method is to partition the interval (a,c) into two halves, namely (a,b) and (b,c), where

$$b = \frac{a+c}{2} \qquad \textbf{3}$$

If "b" is not the solution of equation 1, both partitions are tested further using equation 1 to determine which of the two satisfies equation 2. And the interval containing the root is partitioned further using the procedure above until an approximate root of equation 1 is obtained subject to a pre-determined acceptable error margin. However, an interpolation algorithm is adopted in this thesis to evaluate the root of equation 1 within the acceptable error margin. The interval containing the root, called the neighbourhood N of the root of equation 1 is taken as the best approximation of the root. The iterative procedure is stopped when $\|N\| < \varepsilon$ (where ε is a pre-specified error size.). A typical algorithm used in Bisection method is described below.

Algorithm for the Bisection Method

Having established that the root x of f (x) exists within an interval I = (a, b), the steps involved in the application of the bisection method to find the root of the equation f(x)= 0 are as follows:

1) Choose x_l and x_u within the interval I = (a, b) such that $f(x_l)\,f(x_u) < 0$.
2) Estimate the root, x_m of the equation f (x) = 0 as the mid-point between x_l and x_u as:
$$x_m = \frac{x_l + x_u}{2}$$
3) Now check the followings:
   a) If $f(x_l)\,f(x_m) < 0$, then the root lies between x_l and x_m.
   b) If $f(x_l)\,f(x_m) > 0$, then the root lies between x_m and x_u.
   c) If $f(x_l)\,f(x_m) = 0$, then the root is x_m. Stop the algorithm, ELSE GO TO (d).
   d) Find the new estimate of the root
4) Find the new estimate of the root
$$x_m = \frac{x_l + x_u}{2}$$
and find the absolute relative error as:
$$|\varepsilon_a| = \left|\frac{x^{new} - x^{old}}{x^{new}}\right| \times 100$$
Where:
X^{new} = estimated error from present iteration
X^{old} = estimated error from previous iteration
5) Compare the absolute relative performance error $|\varepsilon_a|$ with the pre-specified relative error tolerance ε_s. If $|\varepsilon_a| > \varepsilon_s$, then go to step 3 or else stop the algorithm.

The advantage of this method is that it will always converge on the root, though the process may be slow because of the process of halving the interval. Also as iterations are conducted, the interval gets halved, so one can guarantee the error in the solution of the equation is minimal.

Appendix 2: Effect of Pulverized bone (PB) on Physical Properties and Setting times of Ordinary Portland Cement mortar

0% PB	SG	W/C Ratio for SC	Setting Times		Retardation RC	
			Initial	Final	Initial	Final
0	2.92	0.30	114	229	0	0
10	2.90	0.28	150	295	36	66
20	2.84	0.28	195	330	81	101
30	2.74	0.27	230	380	16	151

40	2.65	0.26	193	350	79	121
50	2.56	0.24	185	335	71	106
60	2.48	0.23	160	295	46	66
70	0.40	0.23	145	252	31	23
80	0.39	0.22	131	240	17	11
90	0.29	0.21	124	238	10	9
100	2.22	0.20	-	-	-	

Appendix 3: Repeatability of 1600kg/m^3 wet Density Foamed Concrete

%PB	Wet Density	Standard Deviation	Coefficient of Variation
0	1668.28		
5	1627.19		
10	1603.28		
15	1589.69		
20	1563.68		

Appendix 4: Development of Strength-Predicting Model

%PB	k_{SBW}	k_{SBV}	$k_{WS (T)}$	ρ_B	d_c	$A=dc(1+0.20\rho_B + k_{SBV})$	$B=(1+k_{ws})(1+ k_{SBW})\rho_B\gamma_W$	A/B	fc (N/mm^2)
0.00	3.00	3.11	0.172	2.92	1668.3	7566.65	13688.96	0.55	15.43
0.05	3.00	3.11	0.172	2.91	1627.2	7472.06	13642.08	0.54	14.55
0.10	3.00	3.11	0.172	2.90	1603.3	7198.73	13595.20	0.53	13.70
0.15	3.00	3.11	0.172	2.86	1589.7	7124.99	13407.68	0.53	13.20
0.20	3.00	3.11	0.172	2.84	1563.7	7002.16	13313.92	0.52	12.89